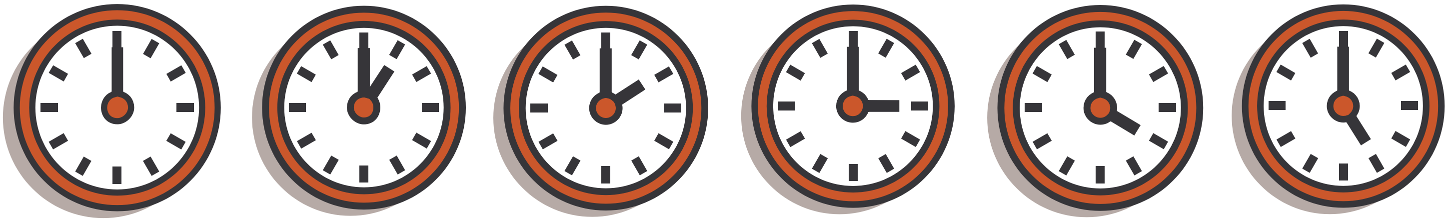

Tracking Time

Representing Elapsed Time on an Open Timeline

Elapsed-time problems are notoriously difficult for children (Monroe, Orme, and Erickson 2002). Instruction on techniques for teaching and learning elapsed time is not emphasized in current mathematics education literature. Nor is it addressed in *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM 2006). This absence of instruction may be due to a position held by some mathematics educators that telling time and determining elapsed time are life skills rather than skills delegated to mathematics instruction. (See Editor's note.) However, time is addressed in the *Principles and Standards for School Mathematics* (NCTM 2000) Measurement Standards for both the pre-K–2 and 3–5 grade bands. And regardless of one's perspective on the delegation of time as a discrete content area, determining elapsed time is encountered during mathematics instruction by most children at some point and is often met with frustration. This is especially true when the start or end time falls between the hour and half hour. Children find it challenging to keep track of unit changes between hours and minutes. On a national assessment, only 58 percent of eighth-grade students were able to correctly identify that 150 minutes equals 2 1/2 hours (Jones and Arbaugh 2004). Elapsed-time instruction often focuses on converting units and keeping track of those conversions rather than on counting up or back from one time to another. This article examines how students are able to make sense of elapsed-time problems when instruction is connected to open number-line strategies. Adapting this technique—typically used for recording addition and subtraction counting strategies—provides a method for supporting students' thinking about elapsed-time problems.

By Juli K. Dixon

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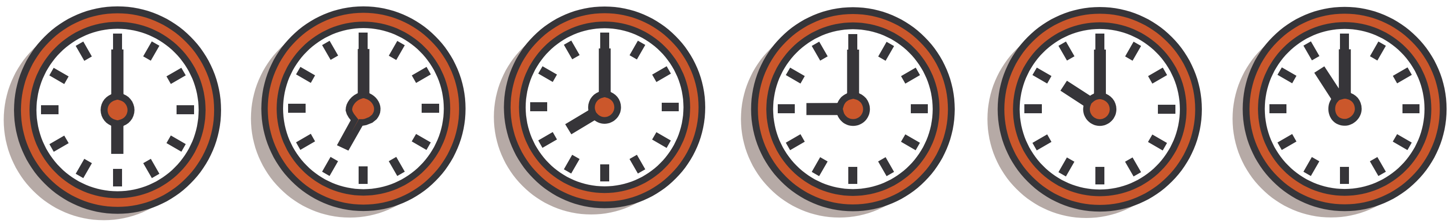
Open Number Lines

The open-number-line strategy is often used to solve whole-number addition and subtraction problems. This strategy is based on counting up, counting back, and finding distances between numbers—explorations consistent with those that can be used for determining elapsed time. Work samples from a third-grade class provide a window into how students can create and evaluate pathways to solve situations involving elapsed time.



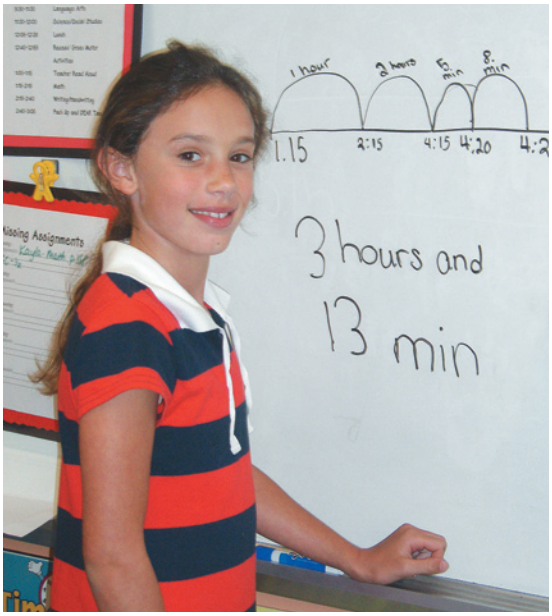


Figure 1

Students generally do not focus on jump "length" in their solutions on the open number line.

36 + 24 = 60

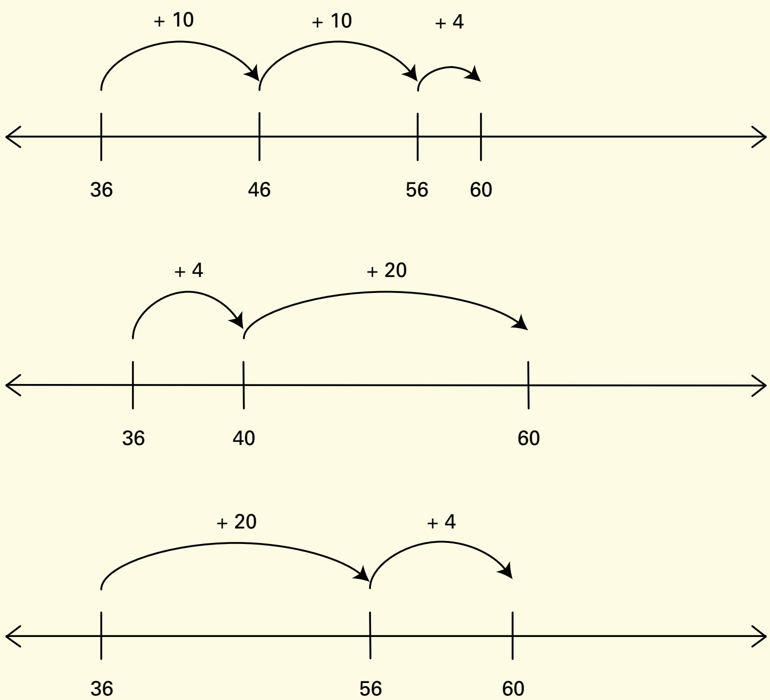
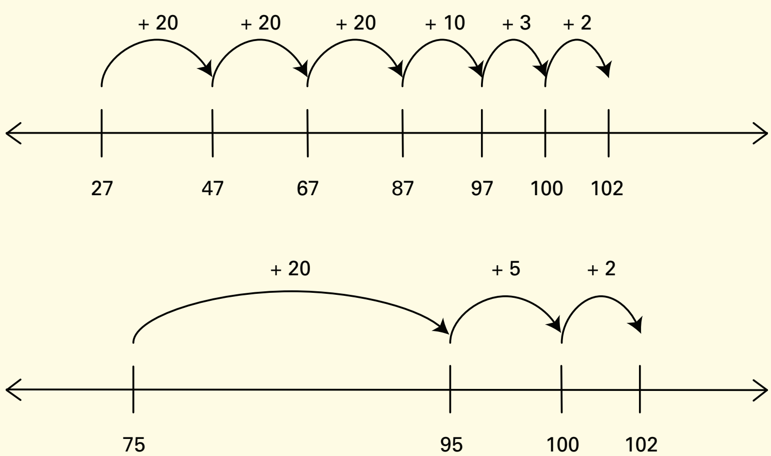


Figure 2

Two strategies to determine the sum when the first addend is smaller than the second

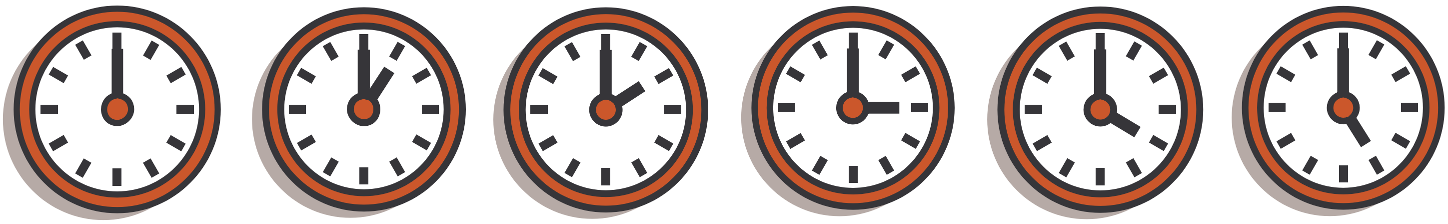
27 + 75 = 102



Addition

When students are presented with a problem such as 36 + 24 and asked to solve it using their own methods, they frequently use counting-on strategies. A common counting-on strategy is to start with one number and add the other number in manageable "chunks." The open number line provides a useful way of recording students' variations on this strategy. Consider the examples in **figure 1**. The first example uses two "jumps" of ten and then a "jump" of four to arrive at the sum of sixty. The second example shows a student adding four to get to forty and then adding another twenty to get to the sum. The third example illustrates first adding twenty in one jump and then adding the remaining four in the second jump. Notice that the equivalent distance between jumps is not necessarily maintained. Sometimes students will leave inconsistent distances between numbers on the number line for greater jumps because they do not focus on this aspect of the problem. In my work with children, I do not address the "length" of the jumps on the number line unless the student initiates it, and even then I allow the student to decide how to proceed.

When the open-number-line strategy is used for problems where the first addend is smaller than the second, such as 27 + 75 (see **fig. 2**), the ways students record their thought processes provide useful assessment data regarding the efficiency of students' strategies. Some students will start from the smaller number and add on the larger number, and others will begin with the larger addend, leaving less to add



on to determine the sum. Students' thinking about their solutions to problems is made explicit as they record their solutions on the number line.

Subtraction

On the open number line, students' solution strategies for adding differ in sophistication and choice of number groupings. However, when subtracting on the open number line, students' solution strategies vary with respect to how they actually approach the problem. Three distinct methods exist for solving subtraction problems (see **fig. 3** for an illustration of each method using the example of 53–37):

- Method 1 is to count up from 37 to 53.
- Method 2 is to count back from 53 to 37.
- Method 3 is to "take away" 37 from 53.

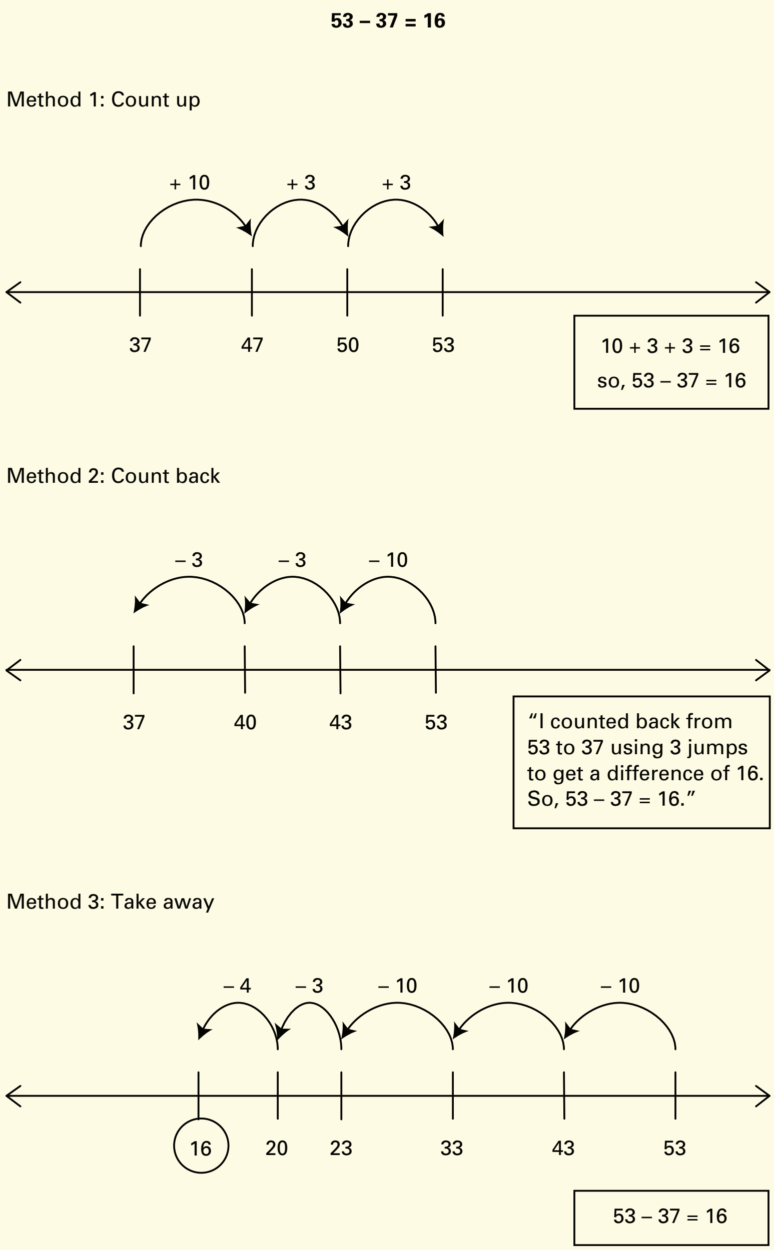
The goal when counting up or counting back is to find the distance between the sum and the given addend. So, for instance, when determining the solution for 53 – 37, students use counting up or counting back strategies to place both 37 and 53 on the number line and then find the distance between the two numbers by "jumping" from one to the other. Using the first method, a student might explain the process by saying—

> I started at thirty-seven and added ten to get to forty-seven. Then I added three so I could get to fifty, but I needed to get to fifty-three, so I added three more. Altogether, I added ten plus three plus three, which equals sixteen. So, fifty-three minus thirty-seven is equal to sixteen.

In the third method, the student begins by placing fifty-three on the number line and then subtracting thirty-seven jumps. The ending number on the number line determines the answer. Two of the three subtraction strategies, counting up and counting back—along with the counting on strategy for addition—describe common ways students can explore elapsed-time situations on the open number line.

Figure 3

Three methods for subtracting on the open number line

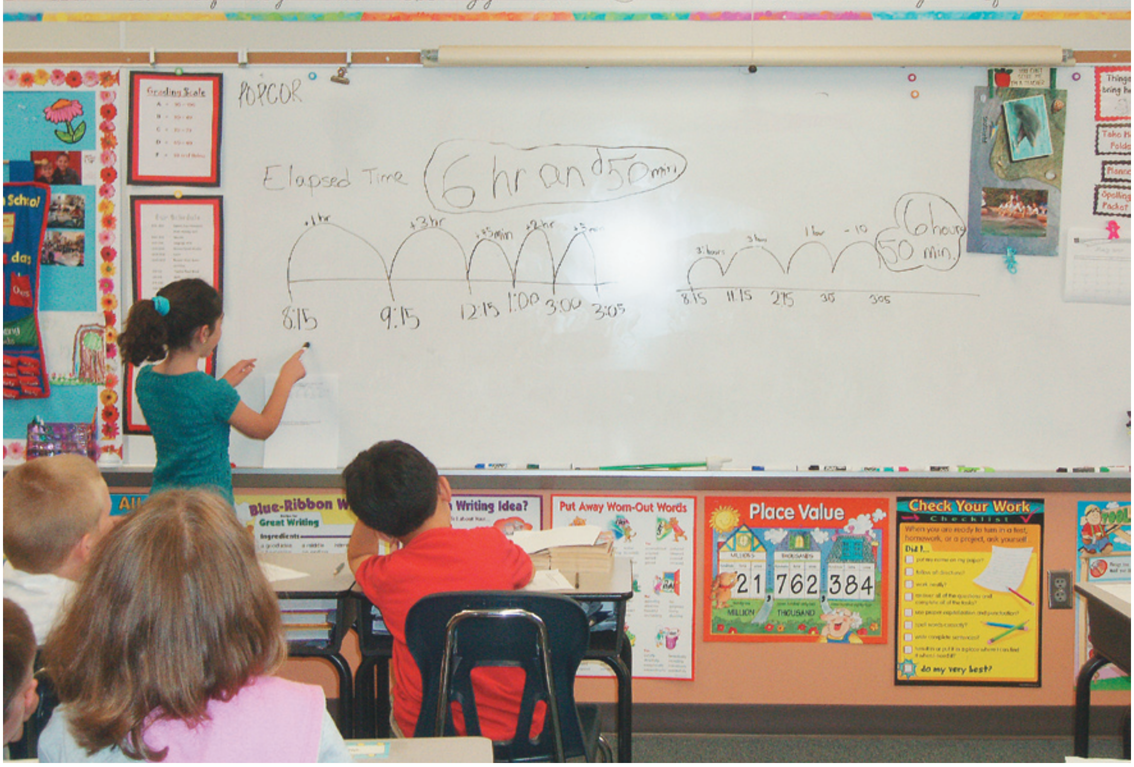


Determining Elapsed Time

When solving elapsed-time problems, students are often first asked to find the time that elapses between two given times. Students typically use the counting up strategy to determine elapsed time. The following word problem is a common type for introducing elapsed time:

> Marni started watching a movie at 3:30. It ended at 4:45. How long did Marni watch the movie?

Students can solve this problem by using counting strategies similar to those used with whole numbers, and their thought processes can be recorded



on an open timeline (see **fig. 4**). A student might describe this process in the following way:

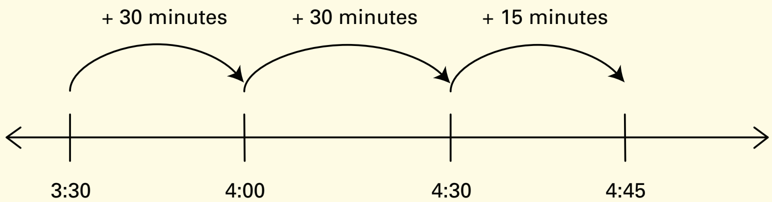
> I started my timeline at 3:30 and added thirty minutes to get to 4:00. I added another thirty minutes to get to 4:30 and fifteen more minutes to get to 4:45. Thirty minutes and thirty minutes is one hour, and then I have fifteen more minutes. Marni watched the movie for one hour and fifteen minutes.

Students are able to use common combinations of time to make hours such as thirty minutes plus thirty minutes or forty minutes plus twenty minutes, and they are also able to use benchmark times like hours, half hours, and quarter hours to help solve problems. Some students find it helpful to count up by hours first until there are no more fully elapsed hours and then count up by minutes to reach the end time. When finding the elapsed time between 1:20 and 4:40, a student might add on one hour three times to go from 1:20 to 2:20 to 3:20 to 4:20 and then add on twenty minutes to get to 4:40. Then students can see that three hours and twenty minutes elapsed between 1:20 and 4:40.

Representing their moves on the timeline facilitates keeping track of elapsed time so that students can make jumps that help them to solve the problem in sense-making ways. The example in **figure 5** provides an illustration of how benchmark times are incorporated in determining elapsed time between 3:40 and 8:15. The solution process involves an initial jump of twenty minutes to get to an hour benchmark. The next jump adds on the full hours remaining between the start and end times. The last jump closes the gap between 8:00 and 8:15. The student is then able to combine jumps in minutes to get

Figure 4

Counting up to determine elapsed time



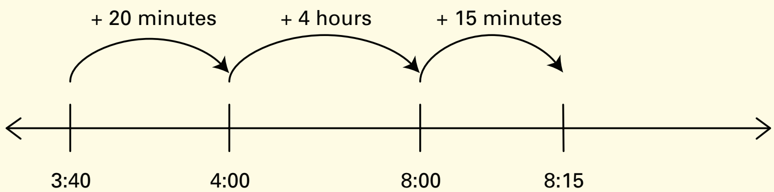
30 min + 30 min = 1 hour
1 hour + 15 min = 1 hour 15 min

Figure 5

Use of hour benchmarks in determining elapsed time

Start time: 3:40

End time: 8:15



20 min + 15 min = 35 min
Elapsed Time: 4 hours 35 minutes

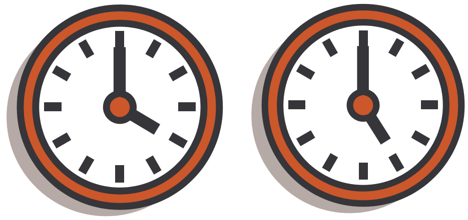


Figure 6

Determining elapsed time that spans the twelve o'clock hour

Start time: 10:30

End time: 2:45

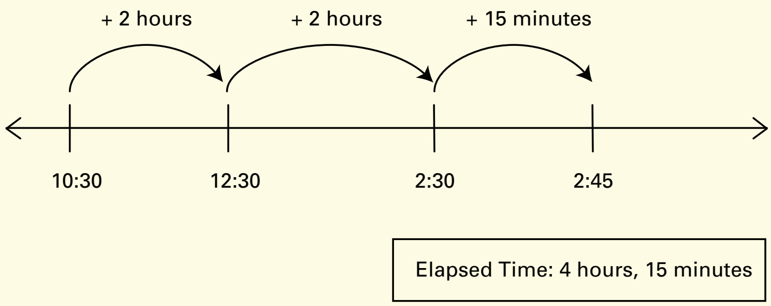


Figure 7

Determining end time by using a jump to a benchmark hour

Start time: 6:55

Elapsed time: 4 hours 30 minutes

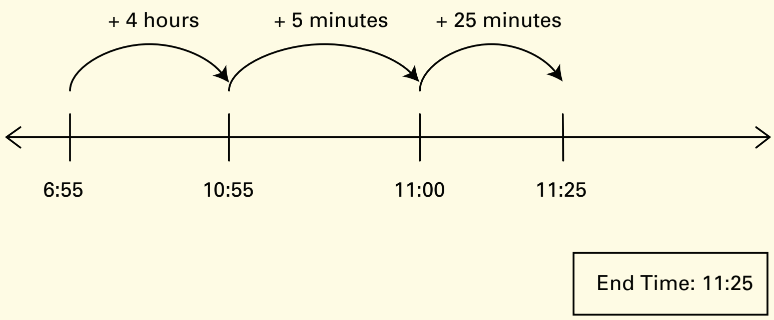
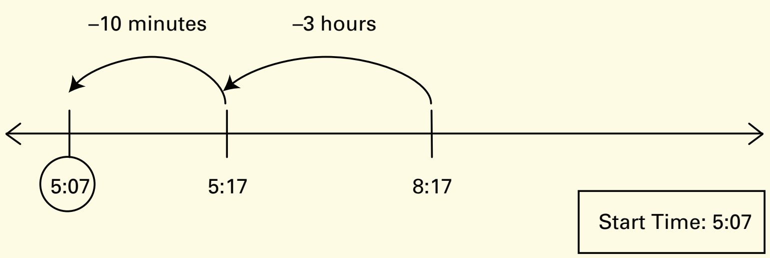


Figure 8

Counting back to determine the start time when the end and elapsed times are given

End time: 8:17

Elapsed time: 3 hours 10 minutes



thirty-five minutes and add that to the four hours for an elapsed time of four hours, thirty-five minutes.

When students have difficulty determining elapsed time that spans over the twelve o'clock hour—such as between 10:30 and 2:45—they can use the timeline to record solutions. With such problems, students typically make a jump that ends in the twelve o'clock hour and continue with jumps to complete the gap in elapsed time (as in **fig. 6**). This process mirrors ways in which adults typically approach such problems in real-life situations.

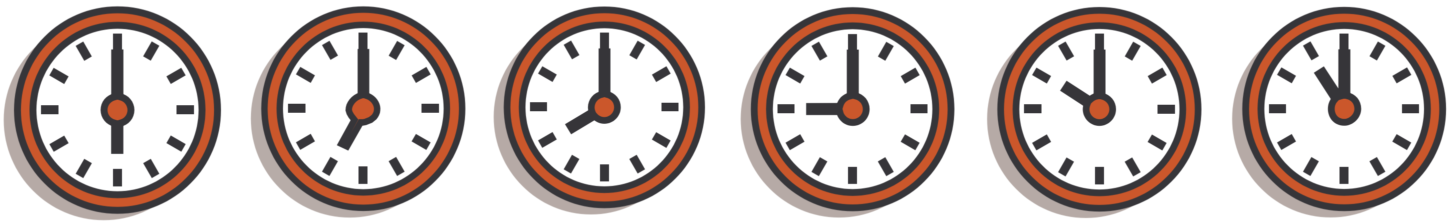
Determining end time from start and elapsed times

When the start and elapsed times are given and students are asked to find the end time, they count on (as was shown with the examples under the Addition heading in this article). When determining end times, the answer is represented below the timeline, whereas the answer to elapsed time is recorded as jumps above the timeline. In **figure 7**, the jumps in elapsed time are recorded above the timeline and broken into sections compatible with using benchmark hours. The start time is 6:55, and the elapsed time is four hours, thirty minutes. The end time is determined by first counting on four hours, from 6:55 to 10:55. The remaining thirty minutes is counted on by breaking the remaining elapsed time into five-minute and twenty-five-minute increments. In this way, the student is able to jump to a benchmark hour of 11:00 before making the final twenty-five-minute jump to the end-time solution of 11:25.

Students generally have no difficulty solving such problems as long as they connect solving elapsed-time problems to working with whole numbers on the open number line. Once your students are successful in determining elapsed and end times using the open timeline, provide them with opportunities to determine start times. These are typically the most difficult elapsed-time problems to solve; however, using the open timeline may make such problems less complicated.

Determining start time from end and elapsed times

In **figure 8**, the end time is not on an interval of five. Examples like these are often avoided because of the perceived complexity involved. However, using the open timeline, students can solve such problems without difficulty. The following example demonstrates how counting back is used to determine the start time when the end time is 8:17 and the elapsed



time is three hours, ten minutes. In this example, a student might start with 8:17, count back three hours to 5:17, and then count back ten minutes to 5:07. The start time is represented under the timeline.

A Third-Grade Open Timeline

Following a brief introduction on how to record solutions to multidigit addition and subtraction problems on the open number line, a class of third graders was presented with a series of elapsed-time problems. Student work was collected and then grouped according to solution strategy (see **figs. 9**, **10**, and **11**).

Students were first asked to find the elapsed time between 8:15 and 3:05 (see **fig. 9**). They solved this problem in several different ways. All of the students counted up from 8:15 to 3:05 by using various jumps. One student jumped by hours to the twelve o'clock hour and then completed the hour by adding forty-five minutes to get to 1:00 (see **fig. 9a**). Several students made jumps to the twelve o'clock hour and then continued to jump the remaining elapsed time.

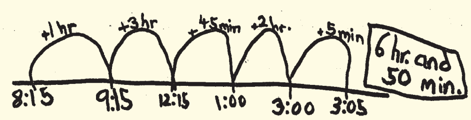
The work samples in **figures 9a** and **b** illustrate how students used both addition and subtraction of time within the same problem to determine elapsed

Figure 9

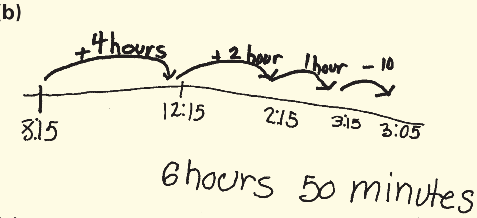
Some third-graders' elapsed-time solutions did not indicate jump directions.

On Monday's, Taylor's school starts at 8:15 and ends at 3:05. How long is she in school on Mondays?

(a)



(b)



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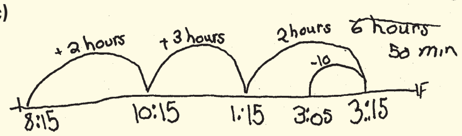
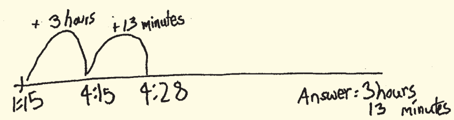


Figure 10

Student solutions to an elapsed-time problem where the end time is not an interval of 5

Alex and Jessica like to sail. On Saturday, they sailed away from the dock at 1:15 and returned at 4:28. How long were they out sailing?

(a)



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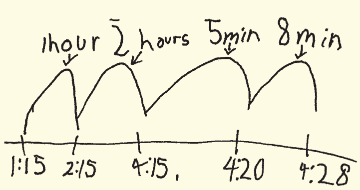

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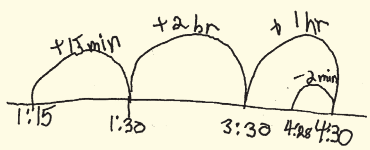


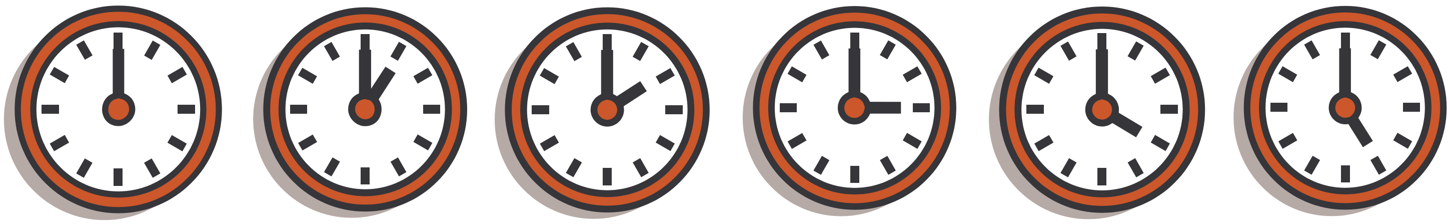
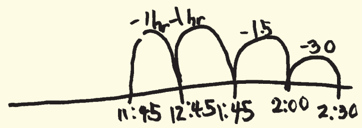


Figure 11

Student solutions to a problem where the start time is unknown

Jake got home from his hike at 2:30 after hiking for 2 hours and 45 minutes. At what time did Jake begin his hike?

(a)



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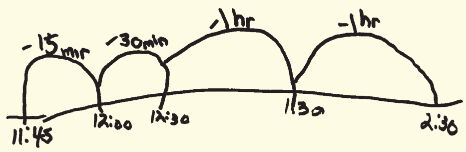

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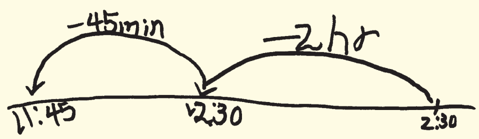

time. These students jumped using complete hours from 8:15 to 3:15 and then "jumped back" ten minutes to end at 3:05. A few students recorded both types of jumps without changing direction on the timeline. Those students were corrected by the students who said that the times you record need to stay in chronological order. Notice that one student did not use any arrows to record the directions of her jumps. Although we modeled and desire this type of notation, we made no attempt to correct students who recorded strategies that made sense to them.

Once students are able to use the open timeline as a tool to determine solutions to problems involving elapsed time, more challenging times and situations can be explored. Similar strategies were used in elapsed-time problems when the time did not end on a multiple of five (see **fig. 10**). When using the open timeline, these problems seemed no more difficult for students than those that started and ended on more common intervals.

Students had some difficulty interpreting problems where the start time was unknown; however, once students made sense of the context of such problems, they were able to solve them with relative ease. **Figure 11** provides students' solutions to a problem where the end time and elapsed time for a hike were given and students were to find the hike's start time. Once students understood that they were to find the hike's start time, they were able to determine that the start time was 11:45.

An Excellent Use of Time

Applying the open-number-line technique for adding and subtracting whole numbers to solve elapsed-time problems is a useful strategy. Students are able to make sense of the problems and record their thought processes as they seek solutions. One of the benefits of the open timeline is that students can use their own solution strategies to solve the problems. They are able to share their timelines with others in the class and provide justifications for their choices. Encourage them to do so. Such opportunities allow students to evaluate their own solution strategies as well as to see how others perceive their solutions. Students benefit from exposure to solution strategies that differ from their own and the chance to make sense of them. Students will see that many approaches to the same problem exist. Provide problems with and without context. Try the problems from this article with your students and see if they solve them in similar ways. You just might find it an excellent use of time.

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Editor's Note: Curriculum Focal Points *delineates the most important mathematical topics for each grade level (pre-K–8). Although* elapsed time *is frequently used to teach an important application of time, the authors of* Curriculum Focal Points *did not consider the topic to be one of the three Focal Points for elementary or middle school students. To learn more about the Focal Points, visit* www.nctm.org/focalpoints.aspx. ▲